(1)

(2) Use branch and bound method to solve the following L.P.P Maximize $Z = 7x_1 + 9x_2$, Subject to the Constraints $-x_1 + 3x_2 \le 6$; $7x_1 + x_2 \le 35$; $x_2 \le 7$
$x_1, x_2 \ge 0$ and are integers.

Solution :

$$-x_1 + 3x_2 = 6$$

when $x_2 = 0$ $\quad -x_1 = 6 \quad x_1 = -6 \quad \therefore (-6, 0)$

when $x_1 = 0$ $\quad 3x_2 = 6 \quad x_2 = 2 \quad \therefore (0, 2)$

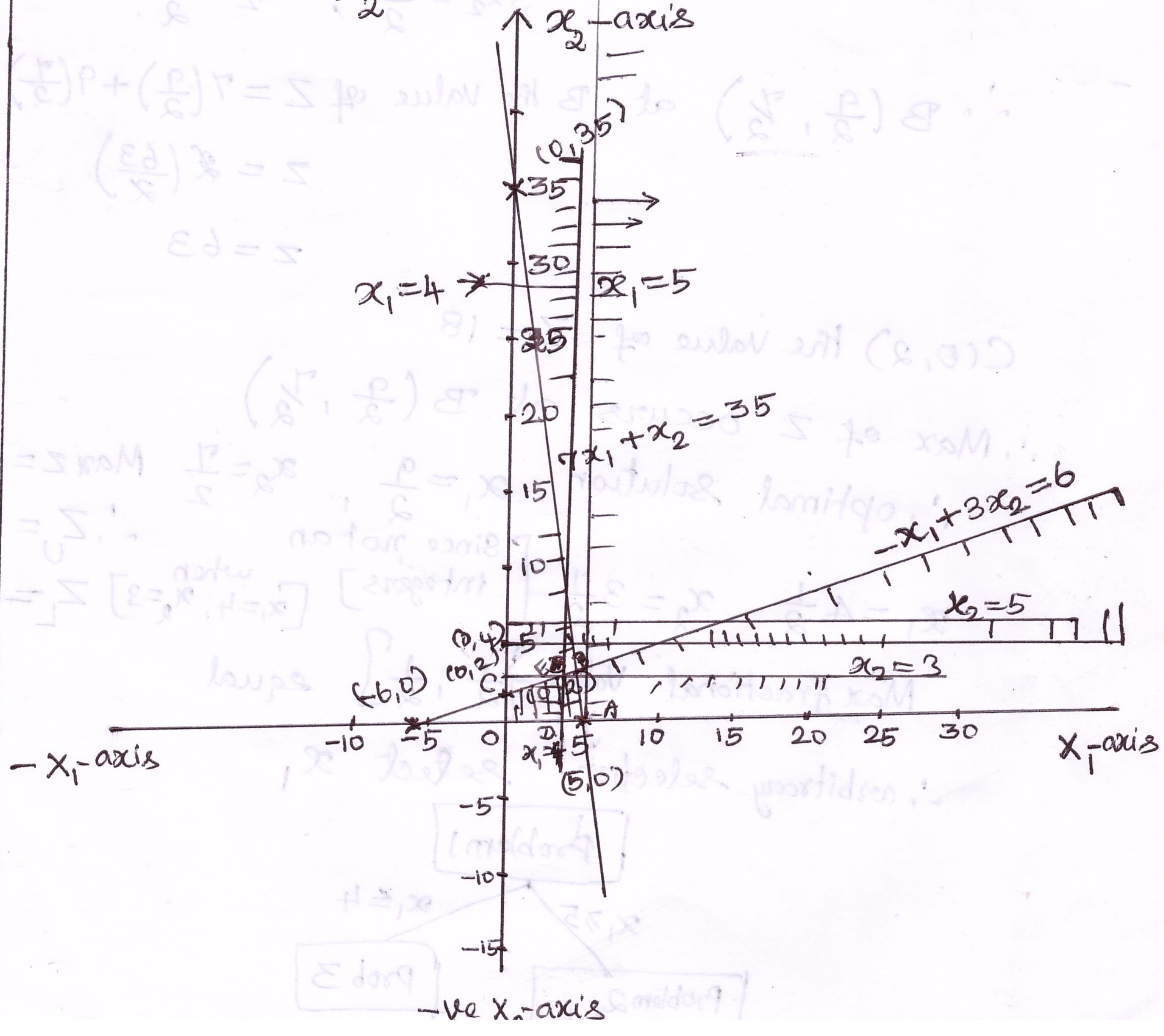
II^{nd} constraint

$$7x_1 + x_2 = 35$$

when $x_2 = 0$ $\quad 7x_1 = 35 \quad x_1 = 5 \quad \therefore (5, 0)$

when $x_1 = 0$ $\quad x_2 = 35 \quad \therefore (0, 35)$

3^{rd} constraint $\quad x_2 = 7$



Feasible region OABC, $Z = 7x_1 + 9x_2$ ②

$O(0,0)$ Value of $Z = 0$

$A(5,0)$ Value of $Z = 35$

B be the point of intersection of the straight lines

$-x_1 + 3x_2 = 6$ — ① and $7x_1 + x_2 = 35$ — ②

① × 1 $\quad -x_1 + 3x_2 = 6$

② × 3 $\quad 21x_1 + 3x_2 = 105$

$$\underline{ - - - }$$

$$-22x_1 = -99 \qquad x_1 = \frac{99}{22} = \frac{9}{2}$$

Sub in ①

$$-\frac{9}{2} + 3x_2 = 6 \qquad 3x_2 = 6 + \frac{9}{2} = \frac{12+9}{2} = \frac{21}{2}$$

$$3x_2 = \frac{21}{2}, \quad x_2 = \frac{7}{2}$$

∴ $B\left(\frac{9}{2}, \frac{7}{2}\right)$ at B the value of $Z = 7\left(\frac{9}{2}\right) + 9\left(\frac{7}{2}\right)$

$$Z = \cancel{2}\left(\frac{63}{\cancel{2}}\right)$$

$$Z = 63$$

$C(0,2)$ the value of $Z = 18$

∴ Max of Z occurs at $B\left(\frac{9}{2}, \frac{7}{2}\right)$

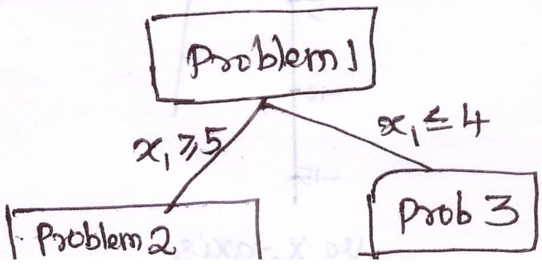
∴ optimal solution $x_1 = \frac{9}{2}$, $x_2 = \frac{7}{2}$ Max $Z = 63$

∴ $Z_U = 63$

$x_1 = 4\frac{1}{2}$ $\quad x_2 = 3\frac{1}{2}$ [since not an integers] when $[x_1 = 4, x_2 = 3]$ $Z_L = 55$

Max fractional Value $\left\{\frac{1}{2}, \frac{1}{2}\right\}$ equal

∴ arbitrary selection select x_1



∴ Problem 2 : feasible region only one point (5, 0)

∴ optimal soln for plbm 2 is $x_1 = 5$ $x_2 = 0$

Max $Z = 35$

Problem 3 Feasible region ODEC $[Z = 7x_1 + 9x_2]$

$O(0, 0)$ at O $Z = 0$

$D(4, 0)$ at D the value of $Z = 28$

E be the point of intersection of $x_1 = 4$ and $-x_1 + 3x_2 = 6$

when $x_1 = 4$ $-4 + 3x_2 = 6$

$3x_2 = 10$

$x_2 = \frac{10}{3}$

∴ $E(4, 10/3)$ the value of Z at E is $Z = 28 + \cancel{9}^{3}\frac{10}{\cancel{3}}$

$Z = \cancel{34}\ 58$

Prob 3

$\boxed{x_1 = 4 \quad x_2 = \frac{10}{3} \quad Z = 58}$ x_2 is not an integer

$x_2 = 3\frac{1}{3}$

$x_2 \geq 4$ $x_2 \leq 3$

Prob 4 No Solution

Prob 5 $x_1 = 4$, $x_2 = 3$ $Z = 55$

Prob4: No feasible region ~~DE~~ No Solution

Prob 5 Feasible region ODFGC $G(3, 3)$ $Z = 48$

$O(0, 0)$ Value of $Z = 0$

$D(4, 0)$ Value of $Z = 28$

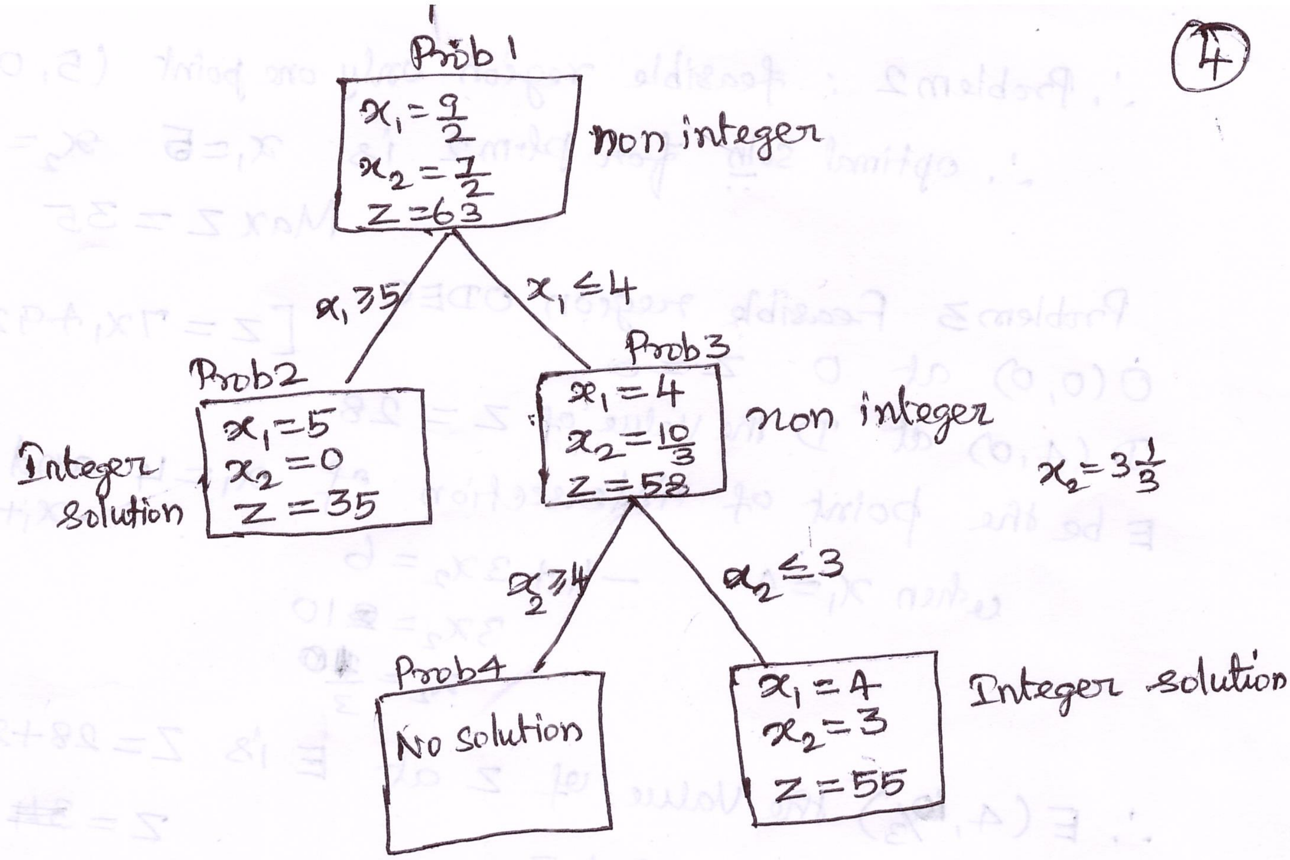
F be the pt of intersection of $x_1 = 4$ $x_2 = 3$

∴ ~~Max~~ Value of $Z = 28 + 27 = 55$

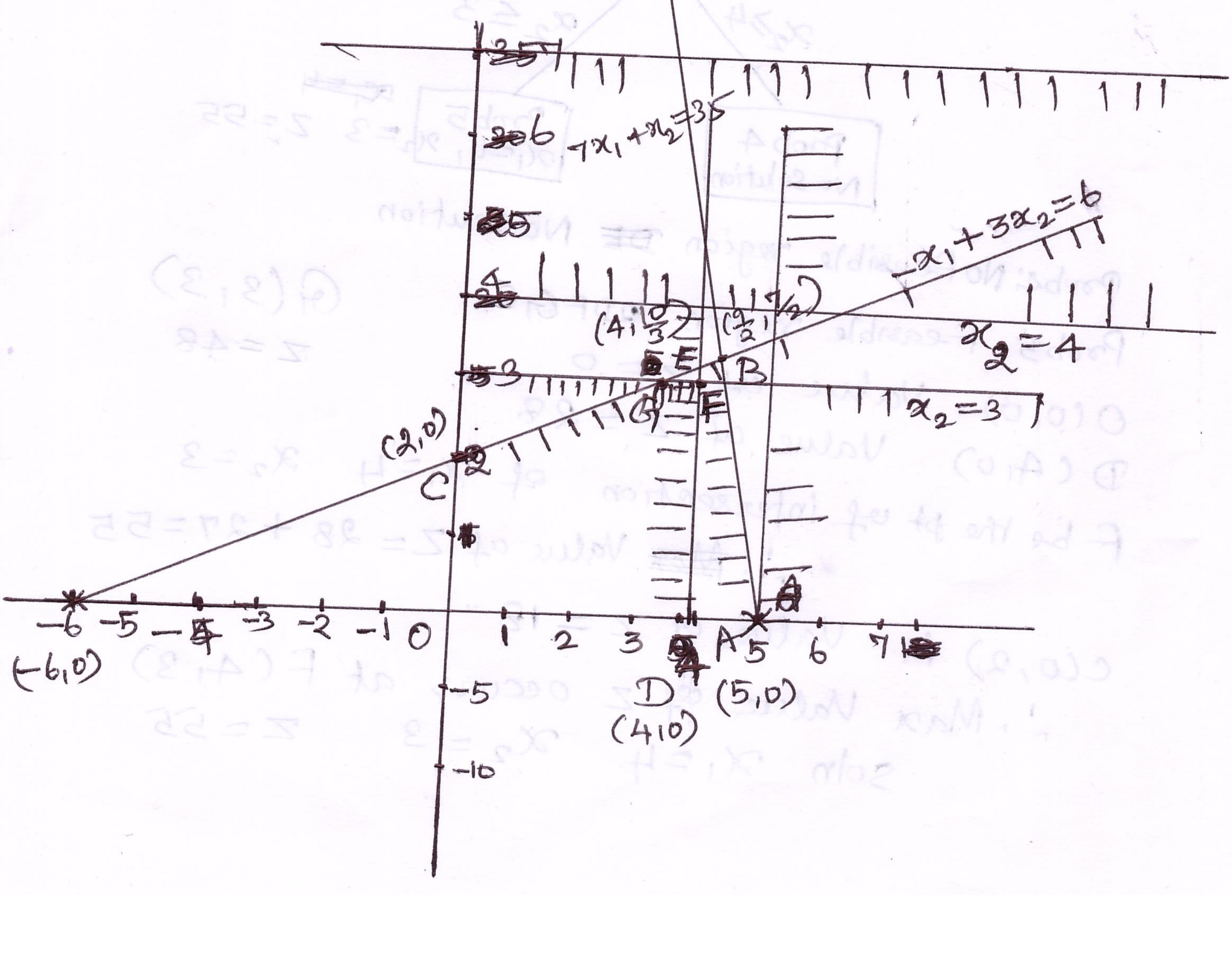
$C(0, 2)$ M. Value of $Z = 18$

∴ Max Value of Z occurs at $F(4, 3)$

soln $x_1 = 4$ $x_2 = 3$ $Z = 55$



$\therefore$ optimal solution $x_1=4$ $x_2=3$ Max $Z=55$



CHAPTER 3

LINEAR PROGRAMMING PROBLEM ARTIFICIAL VARIABLE TECHNIQUE

Introduction:

If any one of the constraint is greater than or equal to sign or equality sign (=). We can use artificial variable, then the method is called artificial variable technique.

Definition 3.1

If any one of the constraint is ≥ sign constraint, to reduce equality sign constraint we have to add one positive variable say A_i in the LHS of the constraint and subtract another positive variable say T_i in the LHS of the constraint, adding variable is called artificial variable, subtracting variable is called surplus variable.

If any one of the constraint is equality sign in that case also we must add one positive variable say A_i in the LHS of the constraint such variable is called artificial variable.

There are two methods of solving LPP using artificial variable

1. Big–M method or Charnes penalty method
2. Two phase simplex method

Method 1 Big-M or Charnes Penalty Method:

Introduction:

Prof.A. Charnes, suggested that a very high penalty should be paid for introducing the artificial variables in the constraints of the given problem, by assigning a very large penalty cost to the artificial variables.

For introducing each artificial variable sustract (MxCorresponding artificial variable) in the R.H.S of the objective function, where M is very large. Therefore the objective function is rewritten as positive value

Maximize $Z = c_1x_1 + c_2x_2 + \dots + c_nx_n - MA_1 - MA_2 - \dots - MA_n$

Where M is very large positive value.

Procedure:

Step1:

Convert the objective function in to maximization if the objective function is minimize Z, then Let $Z^* = -Z$, maximize Z^* subject to the given constraints.

Step2

Reduce standard form of LPP using slack, surplus and artificial variables.

Step3

Introducing each artificial variable subtract M × artificial variable in R.H.S. of the objective function.

Step4

Identify non basic variables and basic variables.

No of Non basic variables = No of var – No of const.

In the beginning take decision variables and surplus variables are in the non basic variable group. Remaining variables are basic variables.

Step5

Find the basic feasible solution.

Let us assume non-basic variables are equal to zero, substitute in the constraint equations we get the values of the basic variables such solution is called basic feasible solution.

Step6

Reduce objective function Z purely interms of non-basic variables and take all the non-basic variables in the LHS

of the objective function.

Step7

Apply usual procedure of simplex method until all the objective function coefficients are positive. That table is called optimal table. Corresponding solution is called optimum solution of the given LPP.

Note

1. In the optimum table, if any one of the artificial variable appears in the basic variable group at non zero level then the given LPP has no feasible solution.
2. If all the coefficients in pivot column are negative or zero then the given LPP has unbounded solution.
3. If any one of the constraint is greater than or equal to sign or equal sign, we must apply either big-M or Two phase Method.

Feasible Solution:

Any solution satisfies all the constraints including non negativity constraints of the given LPP is called feasible solution.

Optimum Solution:

Any feasible solution which optimizes (Maximizes or Minimizes) the objective function of a general LPP is called an optimum solution to the general LPP.

Example3.1.1

Use Penalty Method to Maximize $Z = 3x_1 - x_2$.

Subject to the constraints,

$$2x_1 + x_2 \geq 2$$
$$x_1 + 3x_2 \leq 3$$
$$x_2 \leq 4,$$
$$x_1, x_2 \geq 0.$$

Solution

Reduce Standard Form of LPP

Maximize $Z = 3x_1 - x_2 - MA_1$

Subject to the constraints $2x_1 + x_2 - T_1 + A_1 = 2$

$x_1 + 3x_2 + S_1 = 3$

$x_2 + S_2 = 4$

$x_1, x_2, T_1, A_1, S_1, S_2 \geq 0$

Number of non basic variables = 6 – 3 = 3

Let Non basic variables are x_1, x_2, T_1 and Let $x_1 = x_2 = T = 0$.

$A_1 = 2, S_1 = 3, S_2 = 4.$ Basic feasible Solution

Reduce objective function Z purely in terms of Non Basic variables.

From first constraint $A_1 = 2 - 2x_1 - x_2 + T_1$

$Z = 3x_1 - x_2 - M(2 - 2x_1 - x_2 + T_1)$

$Z = 3x_1 - x_2 - 2M + 2Mx_1 + Mx_2 - MT_1$

$Z = (2M+3)x_1 + (M-1)x_2 - MT_1 - 2M$

$Z - (2M+3)x_1 - (M-1)x_2 + MT_1 = -2M$

Here, M is large positive value $\therefore -(2M+3)$ is most Negative.

1st Iteration

	$x_1 \downarrow$	x_2	T_1	A_1	S_1	S_2	Solution	
Z	$-(2M+3)$	$-(M-1)$	M	0	0	0	$-2M$	
A_1	2	1	–1	1	0	0	2 ←	-2/2=1
S_1	1	3	0	0	1	0	3	3/1=3
S_2	0	1	0	0	0	1	4	

x_1 entering variable and A_1 leaves variable.

2nd Iteration

New Pivot Row = $\dfrac{\text{Old Pivot Row}}{\text{Pivot Element}}$

New Pivot Row = 1, ½,–½,½,0,0,1

New Row=Old Row Element–(Element in the pivot Column ×New Pivot Row)

↓ Pivot column

	x_1	x_2	T_1	A_1	S_1	S_2	Soln
Z	0	$\frac{5}{2}$	$-\frac{3}{2}$	$M+\frac{3}{2}$	0	0	3
x_1	1	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	1
S_1	0	$\frac{5}{2}$	$\frac{1}{2}$ PE	$-\frac{1}{2}$	1	0	2 ← PR
S_2	0	1	0	0	0	1	4

New Pivot row

$\therefore T_1$ entering Variable
S_1 leaves Variable

Z-Coeffts

$= -(2M+3) + (2M+3)\cdot 1 = 0$

$= -(M-1) + (2M+3)\cdot\frac{1}{2} = -\cancel{M} + 1 + \cancel{M} + \frac{3}{2} = \frac{5}{2}$

$= M + (2M+3)(-\frac{1}{2}) = \cancel{M} - \cancel{M} - \frac{3}{2} = -\frac{3}{2}$

$= 0 + (2M+3)(\frac{1}{2}) = M + \frac{3}{2}$

$= 0 + (2M+3)\cdot 0 = 0$

$= 0 + (2M+3)\cdot 0 = 0$

$= -2M + (2M+3)\cdot 1 = 3$

S_1-Coeffts

$= 1 - 1\cdot 1 = 0$

$= 3 - 1(\frac{1}{2}) = \frac{5}{2}$

$= 0 - 1(-\frac{1}{2}) = \frac{1}{2}$

$= 0 - 1(\frac{1}{2}) = -\frac{1}{2}$

$= 1 - 1(0) = 1$

$= 0 - 1(0) = 0$

$= 3 - 1(1) = 2$

	x_1	x_2	T_1	A_1	S_1	S_2	Soln
Z	0	10	0	M	3	0	$\frac{9}{2}$
x_1	1	3	0	0	1	0	$\frac{3}{2}$
T_1	0	5	1	-1	2	0	1
S_2	0	1	0	0	0	1	4

New Pivot row →

all the Z-coeffts are ≥ 0 $\therefore$ above table is optimal, optimal Solution is

$x_1 = \frac{3}{2}$ $x_2 = 0$ Max $z = \frac{9}{2}$.